Let c be any class of *formulae*. We denote by Flg (c) (set of consequences of c) the smallest set of *formulae* which contains all the *formulae* of c and all axioms, and which is closed with respect to the relation "*immediate consequence of*". c is termed ω-consistent, if there is no *class-sign* a such that:

$$(n)\ [Sb\left(a \frac{v}{Z(n)}\right) \varepsilon \text{ Flg } (c)]\ \&\ [\text{Neg}(v \text{ Gen } a)]\ \varepsilon \text{ Flg } (c)$$

where v is the free variable of the *class-sign a*.

Every ω-consistent system is naturally also consistent. The converse, however, is not the case, as will be shown later.

The general result as to the existence of undecidable propositions reads:

Proposition VI: To every ω-consistent recursive class c of *formulae* there correspond recursive class-signs r, such that neither v Gen r nor Neg (v Gen r) belongs to Flg (c) (where v is the free variable of r).

Proof: Let c be any given recursive ω-consistent class of *formulae*. We define:

$Bw_c\ (x) \equiv (n)\ [n \leq l\ (x) \rightarrow A\ x\ (n\ Gl\ x) \vee (n\ Gl\ x)\ \varepsilon\ c \vee$

$(Ep, q)\ \{0 < p, q < n\ \&\ \text{Fl}\ (n\ Gl\ x, p\ Gl\ x, q\ Gl\ x)\}]$

$\&\ l\ (x) > 0$ (5)

(cf. the analogous concept 44)

$x\ B_c y \equiv Bw_c\ (x)\ \&\ [l\ (x)]\ Gl\ x = y$ (6)

$Bew_c\ (x) \equiv (Ey)\ y\ B_c x$ (6.1)

(cf. the analogous concepts 45, 46)

The following clearly hold:

$(x)\ [\text{Bew}_c\ (x) \sim x\ \varepsilon\ \text{Flg}\ (c)]$ (7)

$(x)\ [\text{Bew}\ (x) \rightarrow \text{Bew}_c\ (x)]$ (8)