but to metamathematics, the name given by Hilbert to the study of rigorous proof in mathematics and symbolic logic.

METAMATHEMATICS. Gödel's paper presupposes some knowledge of the state of metamathematics in 1930, which therefore I shall briefly explain. Following on the work of Frege and Peano, Whitehead and Russell's *Principia Mathematica* (1910-13) had exhibited the fundamental parts of mathematics, including arithmetic, as a deductive system starting from a limited number of axioms, in which each theorem is shown to follow logically from the axioms and theorems which precede it according to a limited number of rules of inference. And other mathematicians had constructed other deductive systems which included arithmetic (see p. 37, n. 3). In order to show that in a deductive system every theorem follows from the axioms according to the rules of inference it is necessary to consider the formulae which are used to express the axioms and theorems of the system, and to represent the rules of inference by rules Gödel calls them "mechanical" rules, p. 37) according to which from one or more formulae another formula may be obtained by a manipulation of symbols. Such a representation of a deductive system will consist of a sequence of formulae (a calculus) in which the initial formulae express the axioms of the deductive system and each of the other formulae, which express the theorems, are obtained from the initial formulae by a chain of symbolic manipulations. The chain of symbolic manipulations in the calculus corresponds to and represents the chain of deductions in the deductive system.

But this correspondence between calculus and deductive system may be viewed in reverse, and by looking at it the other way round Hilbert originated metamathematics. Here a calculus is constructed, independently of any interpretation