various technical complications, I believe I am able to give a fairly clear account of everything of a positive nature that can at present be established in connexion with the above problem.

In operating with the metalanguage we shall adhere to the symbolism given in §§ 2 and 3. To simplify the further developments and avoid possible misunderstandings we shall suppose the metalanguage to be so constructed that the language we are studying forms a fragment of it; every expression of the language is at the same time an expression of the metalanguage, but not vice versa. This enables us in certain cases (e.g. in the formulation of condition (α) of convention T) to speak simply of the expressions of the language itself, instead of expressions of the metalanguage which have the same meaning.

After these reservations and conventions we turn to the formulation and proof of the fundamental result.

THEOREM I. (α) *In whatever way the symbol 'Tr', denoting a class of expressions, is defined in the metatheory, it will be possible to derive from it the negation of one of the sentences which were described in the condition (α) of the convention T;*

(β) *assuming that the class of all provable sentences of the metatheory is consistent, it is impossible to construct an adequate definition of truth in the sense of convention T on the basis of the metatheory.*

The idea of the proof of this theorem can be expressed in the following words:[1] (1) a particular interpretation of the meta-

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[1] We owe the method used here to Gödel, who has employed it for other purposes in his recently published work, Gödel, K. (22), cf. especially pp. 174–5 or 187–90 (proof of Th. VI). This exceedingly important and interesting article is not directly connected with the theme of our work—it deals with strictly methodological problems: the consistency and completeness of deductive systems; nevertheless we shall be able to use the methods and in part also the results of Gödel's investigations for our purpose.

I take this opportunity of mentioning that Th. I and the sketch of its proof was only added to the present work after it had already gone to press. At the time the work was presented at the Warsaw Society of Sciences (21 March 1931), Gödel's article—so far as I know—had not yet appeared. In this place therefore I had originally expressed, instead of positive results, only certain suppositions in the same direction, which were based partly on my own investigations and partly on the short report, Gödel, K. (21), which had been published some months previously.

After I had become acquainted with the above mentioned article I convinced myself, among other things, that the deductive theory which Gödel

language is established in the language itself and in this way with every sentence of the metalanguage there is correlated, in one-many fashion, a sentence of the language which is equivalent to it (with reference to the axiom system adopted in the meta-theory); in this way the metalanguage contains as well as every particular sentence, an individual name, if not for that sentence at least for the sentence which is correlated with it and equivalent to it. (2) Should we succeed in constructing in the metalanguage a correct definition of truth, then the metalanguage—with reference to the above interpretation—would acquire that universal character which was the primary source of the semantical antinomies in colloquial language (cf. p. 164). It would then be possible to reconstruct the antinomy of the liar in the metalanguage, by forming in the language itself a sentence x such that the sentence of the metalanguage which is correlated with x asserts that x is not a true sentence. In doing this it would be possible, by applying the diagonal procedure[1] from the theory of sets, to avoid all terms which do not belong to the metalanguage, as well as all premisses of an empirical nature which have played a part in the previous formulations of the antinomy of the liar.[2]

had chosen as the object of his studies, which he called the 'system P', was strikingly similar to the general theory of classes considered in the present section. Apart from certain differences of a 'calligraphical' nature, the only distinction lies in the fact that in the system P, in addition to three logical constants, certain constants belonging to the arithmetic of the natural numbers also occur (a far-reaching analogy also exists between the system P and the system of arithmetic sketched in VI (see pp. 113–16)). Consequently the results obtained for the system P can easily be carried over to the present discussion. Moreover, the abstract character of the methods used by Gödel renders the validity of his results independent to a high degree of the specific peculiarities of the science investigated.

[1] Cf. Fraenkel, A. (16), pp. 48 ff.

[2] If we analyse the sketch of the proof given below we easily note that an analogous reconstruction could be carried out even on the basis of colloquial language, and that in consequence of this reconstruction the antinomy of the liar actually approximates to the antinomy of the expression 'hetero-logical'. For a rather simple reconstruction of the antinomy of the liar in this direction see Tarski, A. (82), note 11, p. 371. It seems interesting that in this reconstruction all the technical devices are avoided which are used in the proof of Th. 1 (such as interpretation of the metalanguage in arithmetic or the diagonal procedure). In connexion with the last paragraph of the text cf. the concluding remarks of § 1, pp. 164 f., and in particular p. 165, note 1.